II. *Æquationum Cubicarum & Biquadraticarum, tum Analytica, tum Geometrica & Mechanica, Resolutio Universalis, a* J. Colson.

§. 1. ÆQuationis Cubicæ Universalis $\left\{ \begin{array}{l} x^3 = 3px^2 + 3qx + 2r, \\ \qquad\qquad -3p^2 \quad + p^3 \\ \qquad\qquad\qquad\quad -3pq \end{array}\right.$

Radices Tres sunt,

$$x = p + \sqrt[3]{r + \sqrt{r^2 - q^3}} + \sqrt[3]{r - \sqrt{r^2 - q^3}}$$

$$x = p - \frac{1 - \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}} - \frac{1 + \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$$

$$x = p - \frac{1 + \sqrt{-3}}{2} \times \sqrt[3]{r + \sqrt{r^2 - q^3}} - \frac{1 - \sqrt{-3}}{2} \times \sqrt[3]{r - \sqrt{r^2 - q^3}}$$

Vel ut Calculus Arithmeticus facilior ac paratior evadat, si posueris Binomii irrationalis $r + \sqrt{r^2 - q^3}$ Radicem Cubicam esse $m + \sqrt{n}$, erunt ejusdem Æquationis Radices tres $x = p + 2m$, & $x = p - m \pm \sqrt{-3n}$.

Igitur data Æquatione quavis Cubica, inter ejus hujusque Æquationis Universalis terminos singulos instituenda est comparatio, quo pacto facillime invenientur ipsæ p, q, r; & hisce cognitis, innotescent Æquationis datæ Radices omnes. Hujus vero Solutionis Exempla sint sequentia in Numeris.

1. Æquationis Cubicæ $x^3 = 2x^2 + 3x + 4$ sit Radix x indaganda. Erit primò juxta præscriptum $3p = 2$,

 sive

five $p=\frac{2}{3}$. Secundò $3q-(3p^2)\frac{4}{3}=3$, five $q=\frac{13}{9}$. Tertiò $2r(+\overline{p^2-3q}\times p)-\frac{70}{27}=4$, five $r=\frac{89}{27}$, & $r^2-q^3=\frac{212}{27}$. Et propterea $x=\frac{2}{3}+\sqrt[3]{\frac{89}{27}+\sqrt{\frac{212}{27}}}+\sqrt[3]{\frac{89}{27}-\sqrt{\frac{212}{27}}}$. Reliquæ duæ Radices funt impoffibiles.

2. In Æquatione $x^3=12x^2-41x+42$, erit primò $3p=12$, five $p=4$. Secundò $3q-(3p^2)48=-41$, five $q=\frac{7}{3}$. Tertiò $2r+(\overline{p^2-3q}\times p)36=42$, five $r=3$; Et inde $r^2-q^3=-\frac{100}{27}$. At Binomii furdi $3+\sqrt{-\frac{100}{27}}(=r+\sqrt{r^2-q^3})$ Radix Cubica, per Methodos ex Arithmetica petendas extracta, eft $-1+\sqrt{-\frac{4}{3}}$, $(=m+\sqrt{n},)$ & proinde Radix $x=(p+2m=4-2=)\,2$, vel etiam $x=(p-m\pm\sqrt{-3n}=4+1\pm(\sqrt{4})\,2=)\,7$ vel 3. Vel rurfus, ejufdem Binomii $3+\sqrt{-\frac{100}{27}}$. Radix alia Cubica (tres enim agnofcit) eft $\frac{3}{2}+\sqrt{-\frac{1}{12}}$ $(=m+\sqrt{n},)$ & proinde Radix $x=(p+2m=4+3=)\,7$, & etiam $x=(p-m\pm\sqrt{-3n}=4-\frac{3}{2}\pm(\sqrt{\frac{1}{4}})\frac{1}{2}=3$ vel 2. Vel denuo, ejufdem Binomii $3+\sqrt{-\frac{100}{27}}$ Radix Cubica tertia eft $-\frac{1}{2}-\sqrt{-\frac{25}{12}}$, $(=m+\sqrt{n},)$ & proinde Radix

$x=$

$x = (p + 2m = 4 - 1 =) 3$, atque etiam $x = (p - m \pm \sqrt{-3n} = 4 + \frac{1}{2} \pm (\sqrt{\frac{25}{4}}) \frac{5}{2} =) 7$ vel 2.

3. In Æquatione $x^3 = -15x^2 - 84x + 100$, erit $p = -5$, $q = -3$, $r = 135$; & Binomii $135 + \sqrt{18252}$ Radix Cubica est $3 + \sqrt{12}$. Igitur Radix $x = -5 + 6 = 1$, & $x = -5 - 3 \pm \sqrt{-36} = -8 \pm \sqrt{-36}$, impossibiles.

4. In Æquatione $x^3 = 34x^2 - 310x + 1012$, erit $p = \frac{34}{3}$, $q = \frac{226}{9}$, $r = \frac{5536}{27}$; & Binomii $\frac{5536}{27} + \sqrt{\frac{707560}{27}}$ Radix Cubica est $\frac{16}{3} + \sqrt{\frac{10}{3}}$. Igitur Radix $x = \frac{34}{3} + \frac{32}{3} = 22$, & $x = \frac{34}{3} - \frac{16}{3} \pm \sqrt{-10} = 6 \pm \sqrt{-10}$, impossibiles.

5. In Æquatione $x^3 = 28x^2 + 61x - 4048$, erit $p = \frac{28}{3}$, $q = \frac{967}{9}$, $r = -\frac{25010}{27}$; & Binomii $-\frac{25010}{27} + \sqrt{-382347}$. Radix Cubica est $\frac{41}{6} + \sqrt{-\frac{243}{4}}$. Igitur $x = \frac{28}{3} + \frac{41}{3} = 23$, & $x = \frac{28}{3} - \frac{41}{6} \pm (\sqrt{\frac{729}{4}}) \frac{27}{2} = 16$ vel -11.

6. In Æquatione $x^3 = -x_2 + 166x - 660$, erit $p = -\frac{1}{3}$, $q = \frac{499}{9}$, $r = -\frac{9658}{27}$; & Binomii $-\frac{9658}{27} + \sqrt{-\frac{1147205}{27}}$ Radix Cubica est $-\frac{22}{3} + \sqrt{-\frac{5}{3}}$. Igitur $x = -\frac{1}{3} - \frac{44}{3} = -15$, & $x = -\frac{1}{3} + \frac{22}{3} \pm \sqrt{5} = 7 \pm \sqrt{5}$, irrationales.

7. In

7. In Æquatione $x^3 = 63x^2 + 99673x + 9951705$, erit $p = 21$, $q = \frac{100996}{3}$, $r = 6031680$; & Binomii $6031680 + \sqrt{-\frac{47887175043136}{27}}$ Radix Cubica est $183 + \sqrt{-\frac{529}{3}}$. Igitur $x = 21 + 366 = 387$, & $x = 21 - 183 \pm (\sqrt{529})23 = -139$ vel 185.

Nec secus in cæteris procedendum: Investigatur autem Theorema ad modum sequentem. Pono Æquationis cujusdam Cubicæ Radicem $z = a + b$, & cubicè multiplicando proveniet $z^3 = (a^3 + 3a^2b + 3ab^2 + b^3 =)$ $a^3 + 3ab \times \overline{a + b} + b^3$. Jam loco ipsius $a + b$ valorem ejus z substituendo, fiet $z^3 = 3abz + a^3 + b^3$, quæ est Æquatio Cubica ex Radice $z = a + b$ constructa, cui terminus secundus deest. Ut hæc verò ad formam magis commodam magisq; concinnam revocenter, sumo Æquationem $z^3 = 3qz + 2r$, quæ posthac ipsius $z^3 = 3abz + a^3 + b^3$ vices gerat. Igitur transmuratione hujus in illam, fiet primò $3q = 3ab$, sive $q_3 = a^3b^3$; & secundò $2r = a^3 + b^3$, sive $2ra^3 = (a^6 + a^3b^3 =) a^6 + q^3$. Et soluta hac æquatione quadratica, erit $a^3 = r + \sqrt{r^2 - q^3}$, & hinc $b^3 = (2r - a^3 =) r - \sqrt{r^2 - q^3}$: Atque igitur tandem $a = \sqrt[3]{r + \sqrt{r^2 - q^3}}$ & $b = \sqrt[3]{r - \sqrt{r^2 - q^3}}$. Et propterea in Æquatione Cubica $z^3 = 3qz + 2r$ erit Radix $z = (a + b =) \sqrt[3]{r^2 + \sqrt{r - q^3}} + \sqrt[3]{r^2 - \sqrt{r - q^3}}$

At verò hæc Radix reverà triplex est, pro triplici valore quem induere potest & $\sqrt[3]{r + \sqrt{r^2 - q^3}}$ & $\sqrt[3]{r - \sqrt{r^2 - q^3}}$. Cujusvis enim quantitatis Radix Cubica triplex erit, & ipsius Unitatis Radix Cubica vel

est

est 1, vel $-\frac{1}{2}+\frac{1}{2}\sqrt{-3}$, vel $-\frac{1}{2}-\frac{1}{2}\sqrt{-3}$: Atque id adeo, propterea quòd harum alicujus Cubus sit Unitas. Igitur si $1\times\sqrt[3]{r+\sqrt{r^2-q^3}}$ aut $\sqrt[3]{r+\sqrt{r^2-q^3}}$ ($=\sqrt[3]{1\times r+\sqrt{r^2-q^3}}=\sqrt[3]{1}\times\sqrt[3]{r+\sqrt{r^2-q^3}}$) Radicem aliquam [quam supra nominavimus $m+\sqrt{n}$, aut $1\times\overline{m+\sqrt{n}}$,] Cubi $r+\sqrt{r^2-q^3}$ designet; ipsæ $\frac{-1+\sqrt{-3}}{2}\times\sqrt[3]{r+\sqrt{r^2-q^3}}$ & $\frac{-1-\sqrt{-3}}{2}\times\sqrt[3]{r+\sqrt{r^2-q^3}}$ [i. e. $\frac{-1+\sqrt{-3}}{2}\times\overline{m+\sqrt{n}}$ & $\frac{-1-\sqrt{-3}}{2}\times\overline{m+\sqrt{n}}$] alias duas ejusdem Cubi Radices designabunt. Similiter & $\sqrt[3]{r-\sqrt{r^2-q^3}}$, $\frac{-1+\sqrt{-3}}{2}\times\sqrt[3]{r-\sqrt{r^2-q^3}}$, & $\frac{-1-\sqrt{-3}}{2}\times\sqrt[3]{r-\sqrt{r^2-q^3}}$, [i. e. $m-\sqrt{n}$, $\frac{-1+\sqrt{-3}}{2}\times\overline{m-\sqrt{n}}$, $\frac{-1-\sqrt{-3}}{2}\times\overline{m-\sqrt{n}}$,] tres Cubicæ Radices erunt Apotomes $r-\sqrt{r^2-q^3}$. Atque has Radices debitè connectendo, fiet $z=\sqrt[3]{r+\sqrt{r^2-q^3}}+\sqrt[3]{r-\sqrt{r^2-q^3}}$, [i. e. $z=m+\sqrt{n}+m-\sqrt{n}=2m$,] $z=\frac{-1+\sqrt{-3}}{2}\times\sqrt[3]{r+\sqrt{r^2-q^3}}+\frac{-1-\sqrt{-3}}{2}\times\sqrt[3]{r-\sqrt{r^2-q^3}}$, [i. e. $z=\frac{-1+\sqrt{-3}}{2}\times\overline{m+\sqrt{n}}+\frac{-1-\sqrt{-3}}{2}\times\overline{m-\sqrt{n}}=-m+\sqrt{-3n}$,] & $z=\frac{-1-\sqrt{-3}}{2}\times\sqrt[3]{r+\sqrt{r^2-q^3}}+\frac{-1+\sqrt{-3}}{2}\times\sqrt[3]{r-\sqrt{r^2-q^3}}$

[i. e. $z = \frac{-1-\sqrt{-3}}{2} \times \overline{m+\sqrt{n}} + \frac{-1+\sqrt{-3}}{2} \times \overline{m-\sqrt{n}} = -m - \sqrt{-3\,n}$,] quæ tres erunt Radices Æquationis Cubicæ $z^3 = 3\,q\,z + 2\,r$. Debite autem connectuntur Radices istæ ad modum præcedentem, quippe quæ sic connexæ, & more vulgari in se invicem continue ductæ, Æquationem $z^3 = 3\,q\,z + 2\,r$ restituunt. Denique fac $z = x - p$, & Æquatio fiet $x^3 - 3\,p\,x^2 + 3\,p^2x - p^3 = 3\,q\,x \quad 3\,p\,q + 2\,r$, quæ universalis est, & cujus Radices evadunt ut supra fuerunt exhibitæ.

Hic obiter notatu dignum est, quod Æquationis Cubicæ cujuscunque Radices omnes sint possibiles & reales, quoties Binomii membrum irrationale $\sqrt{r^2 - q^3}$ impossibilitatem in se complectitur; hoc est, quoties q est quantitas affirmativa, & simul cubus ejus major est quadrato ex latere r. At si membrum istud $\sqrt{r^2 - q^3}$ sit possibile, hoc est si q sit quantitas negativa, aut etiam si affirmativæ cubus sit minor quadrato ex latere r, tunc unicam tantùm agnoscit Æquatio Radicem possibilem & realem, reliquæque duæ erunt impossibiles.

In hoc Theoremate si fiat $p = 0$, hoc est, si desit Æquationis terminus secundus, tunc deventum erit ad casum Regularum quæ dicuntur *Cardani*, cujus solutio continetur in præcedentibus.

§. 2. Æquationis Biquadraticæ Universalis

$$x^4 = 4\,p\,x^3 + 2\,q\,x^2 + 8\,r\,x + 4s, \\ \qquad\qquad - 4\,p^2 \quad - 4pq \quad - q^2$$

Radices quatuor sunt $x = p - a \pm \sqrt{p^2 + q - a^2 - \frac{2r}{a}}$,

& $x = p + a \pm \sqrt{p^2 + q - a^2 + \frac{2r}{a}}$, Ubi a^2 est Radix Æquationis Cubicæ

$$a^6 = \begin{matrix} p^2 \\ +q \end{matrix}\, a^4 \begin{matrix} -2\,p\,r \\ -s \end{matrix}\, a^2 + r^2.$$

Jam data Æquatione quavis Biquadratica, inter ejus hujusque Æquationis Universalis terminos singulos instituenda

enda eſt comparatio, quo pacto citiſſime invenientur ipſæ p, q, r, s; & hiſce cognitis, non latebit valor ipſius a, ex Theoremate ſuperiori inveniendus, & tum demum innoteſcent Æquationis datæ Radices omnes.

Huic Solutioni illuſtrandæ Exemplum unum aut alterum ſufficiat.

1. Æquationis Biquadraticæ $x^4 = 8x^3 + 83x^2 - 162x - 936$ ſint Radices extrahendæ. Erit primò juxta præſcriptum $4p = 8$, ſive $p = 2$. Secundò $2q - (4p^2)$ $16 = 83$, ſive $q = \frac{99}{2}$. Tertiò $8r - (4pq)\ 396 = -162$, ſive $r = \frac{117}{4}$. Quartò $4s - (q^2)\ \frac{9801}{4} = -936$, ſive $s = \frac{6057}{16}$. Hinc $p^2 + q = \frac{107}{2}$, $2pr + s = \frac{7929}{16}$, $r^2 = \frac{13689}{19}$, & proinde $a^6 = \frac{107}{2}a^4 - \frac{7929}{16}a^2 + \frac{13689}{16}$. Jam ut Æquatio hæc aliquatenus Cubica in Radices ejus reſolvatur, ad Theorema præcedens recurrendum eſt, in quo erit $p = \frac{107}{2}$, $q = \frac{22009}{144}$, $r = \frac{2903923}{1728}$ & $r^2 - q^3 = -\frac{11940075}{16}$. Atqui Binomii $\frac{2903923}{1728} + \sqrt{-\frac{11940075}{16}}$ Radix Cubica eſt $-\frac{53}{12} + \sqrt{-\frac{400}{3}}$ & propterea $a^2 = \frac{107}{6} - \frac{53}{6} = 9$, & etiam $a^2 = \frac{107}{6} + \frac{53}{12} \pm (\sqrt{400})\ 20 = \frac{169}{4}$ vel $\frac{9}{4}$: Vel quod perinde eſt, Æquationis præmiſſæ reverà Cubo-Cubicæ ſex Radices ſunt $a = \pm 3$, $a = \pm \frac{13}{2}$, & $a = \pm \frac{3}{2}$, quarum quævis indiſcriminatim propo-

ſito

sito nostro faciet satis. Puta si in præsenti casu fiat $a = 3$, erit juxta Theorema $x = (p - a \pm \sqrt{p^2 + q - a^2 - \frac{2r}{a}} = 2 - 3 \pm \sqrt{4 + \frac{99}{2} - 9 - \frac{39}{2}} = -1 \pm (\sqrt{25})\, 5 =)\, 4$ vel -6, & $x = (p + a \pm \sqrt{p^2 + q - a^2 + \frac{2r}{a}} = 2 + 3 \pm \sqrt{4 + \frac{99}{2} - 9 + \frac{39}{2}} = 5 \pm (\sqrt{64})\, 8 =)\, 13$ vel -3, quæ sunt Æquationis datæ Radices quatuor.

2. In Æquatione $x^4 = 20x^3 + 252x^2 - 6592x + 21312$, erit $p = 5$, $q = 176$, $r = -384$, & $s = 13072$. Hinc $p^2 + q = 201$, $2pr + s = 9232$, & $r^2 = 147456$; & inde $a^6 = 201\, a^4 - 9232\, a^2 + 147456$. Jam in Theoremate pro Cubicis erit $p = 67$, $q = \frac{4235}{3}$, & $r = 65219$; eritque Binomii $65219 + \sqrt{\frac{38889307072}{27}}$ Radix Cubica $\frac{77}{2} + \sqrt{\frac{847}{12}}$. Igitur $a^2 = 67 + 77 = 144$, sive $a = 12$; & proinde $x = 5 - 12 \pm \sqrt{25 + 176 - 144 + 64} = -7 \pm (\sqrt{121})\, 11 = 4$ vel -18, & $x = 5 + 12 \pm \sqrt{25 + 176 - 144 - 64} = 17 \pm \sqrt{-7}$, impossibiles.

Hujus autem Theorematis Inventio est hujusmodi. Ex duarum Æquationum Quadraticarum $z^2 + 2az - b = 0$, & $z^2 - 2az - c = 0$ in se invicem multiplicatione, Æquationem conficio Biquadraticam $z^4 = \overline{4a^2 + b + c} \times z^2 + \overline{2ac - 2ab} \times z - bc$, cui terminus secundus deest, quamque huic Æquationi $z^4 = ez^2 + fz + g$ statuo æquipollere. Unde primo $4a^2 + b + c = e$ sive $b = e - 4a^2 - c$. Secundò $2ac - 2ab = f$, hoc est, $2ac - 2ae + 8a^3 + 2ac = f$, sive $c = \frac{f}{4a} + \frac{e}{2} - 2a^2$,

&

& inde $b = (e - 4a^2 - c =) - \frac{f}{4a} + \frac{e}{2} - 2a^2$. Tertiò $-bc = g$, ſive $-\frac{f^2}{16a^2} + \frac{e^2}{4} - 2ea^2 + 4a^4 = -g$, hoc eſt, $a^6 = \frac{1}{2}ea^4 - \frac{1}{4}ga^2 - \frac{1}{16}ea^2 + \frac{f^2}{64}$, quæ Æquatio quaſi Cubica eſt, ex Radice a^2 & notis vel aſſumptis e, f, g conflata. Ea verò Radix per Theorema ſuperius exhiberi poteſt, & eodem Calculo innoteſcent ipſæ b & c. At Æquationum $z^2 + 2az - b = o$ & $z^2 - 2az - c = o$ Radices ſunt $z = -a \pm \sqrt{a^2 + b}$ & $z = a \pm \sqrt{a^2 + c}$, ſive $z = -a \pm \sqrt{\frac{1}{2}e - a^2 - \frac{f}{4a}}$, & $z = a \pm \sqrt{\frac{1}{2}e - a^2 + \frac{f}{4a}}$, quæ proinde erunt Radices Æquationis $z^4 = ez^2 + fz + g$; cognita videlicet a vel a^2 ex Æquatione $a^6 = \frac{1}{2}ea^4 - \frac{1}{4}ga^2 - \frac{1}{16}ea^2 + \frac{f^2}{64}$. Jam ut Æquatio iſta evadat univerſalis, & omnibus ſuis terminis inſtructa, fac. $z = x - p$, eritque $x^4 - 4px^3 + 6p^2x^2 - 4p^3x + p^4 = ex^2 - 2pex + p^2e + fx - fp + g$, item & $x = p - a \pm \sqrt{\frac{1}{2}e - a^2 - \frac{f}{4a}}$, & $x = p + a \pm \sqrt{\frac{1}{2}e - a^2 + \frac{f}{4a}}$. Tandem concinnitatis & compendii gratiâ, fac $e = 2q + 2p^2$ & $f = 8r$; tum $x^4 - 4px^3 + 4p^2x^2 = 2qx^2 - 4pqx + 2p^2q + p^4 + 8rx - 8pr + g$, $x = p - a \pm \sqrt{p^2 + q - a^2 - \frac{2r}{a}}$, $x = p + a \pm \sqrt{p^2 + q - a^2 + \frac{2r}{a}}$, & $a^6 = \overline{p^2 + q} \times a^4 - \overline{\frac{1}{4}g + \frac{1}{4}p^4 + \frac{1}{2}p^2q \;\; \frac{1}{4}q^2} \times a^2 + r^2$. Denique fac $g = 4s - q^2 + 8pr - p^4 - 2p^2q$, & fiunt Æquationes præcedentes

$$x^4 = 4px^3 + \begin{matrix}2q\\-4p^2\end{matrix}x^2 + \begin{matrix}8r\\-4pq\end{matrix}x + \begin{matrix}4s\\-q^2\end{matrix} \quad \& \quad a^6 = \begin{matrix}p^2\\+q\end{matrix}a^4 - \begin{matrix}2pr\\-s\end{matrix}a^2 + r^2.$$

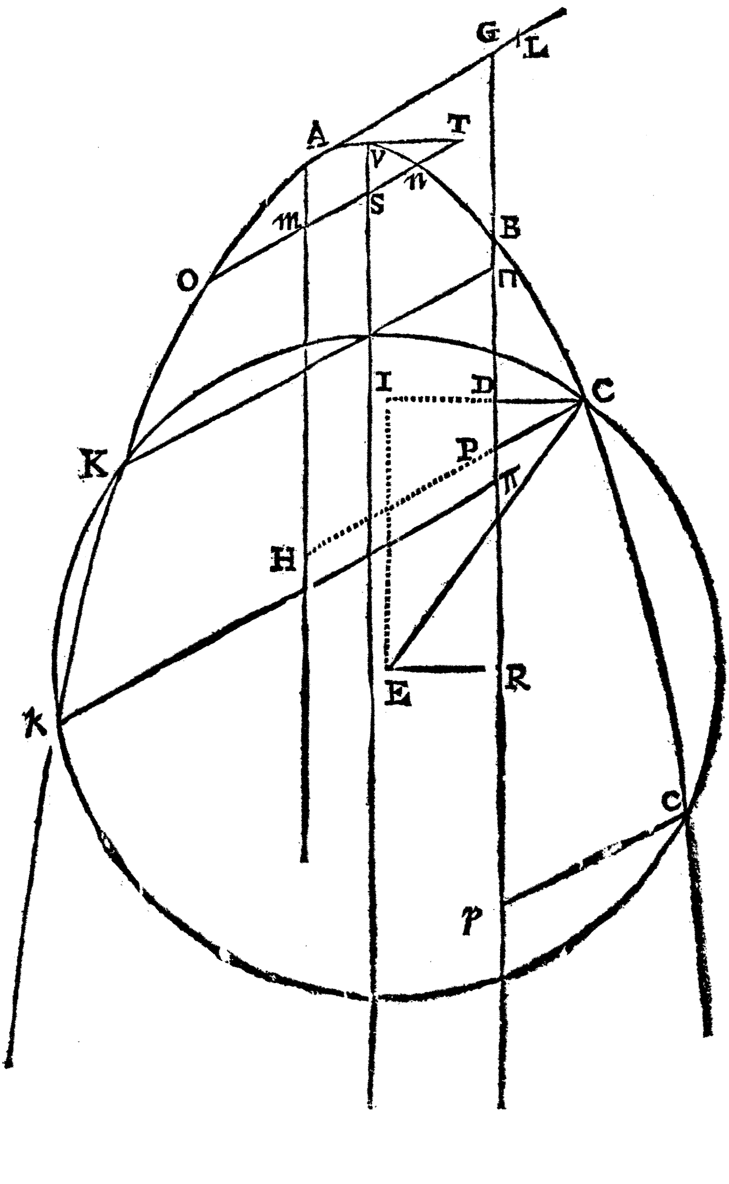
Scilicet omnia evadunt ut ſupra ſunt poſita.

§ 8. Hactenus de Æquationum Cubicarum & Biquadraticarum Resolutione Analytica. Quoniam autem earundem *Effectio Geometrica* per Parabolam vulgò tradi solet, & nonnullis in pretio est, ipsam συνοπτικῶς, & quidem universalius, non pigebit hic exhibere.

Data Æquatione quavis vel Cubica vel Biquadratica, instituenda est comparatio inter terminos ejus, terminosque respondentes hujus Æquationis

$$\begin{array}{llllll} x^4 = & \frac{2p}{q} x^3 & + \frac{4pr}{q} x^2 & + \frac{2pz}{q} x & + p^2, & \text{quo pacto facile satis} \\ & -4r & -4r^2 & -\frac{2ps}{q} & -q^2 & \\ & & +2s & +4rs & -s^2 & \\ & & -1 & -2q & +t^2 & \end{array}$$

eruentur ipsæ p, q, r, s, t; earum interim unâ aliquâ utcunque pro lubitu assumptâ. Tum in Parabola quavis data AVB, cujus Vertex principalis V, Axis VS, & Axi



 per-

perpendicularis VT; capiatur VS $\div$ p versus interiora Parabolæ, & in angulo SVT inscribatur ST = q, quæ producta Parabolam secet in punctis binis N & O. Bisecetur ON in M, & per M agatur MA Axi parallela & Parabolæ occurrens in A. Ipsi ON parallela ducatur AL, ut sit AL Latus rectum Parabolæ ad Diametrum AM, sitque hæc eadem Unitas. In AL (utrinque si opus est producta) capiatur AG = r, & à puncto G ducatur GR Axi parallela, quæ Parabolam secet in B, à quo capiatur BR = s. A novissime invento puncto R ducatur RE ipsi VT parallela & æqualis, quæ sinistram versus jaceat respectu ipsius R si q sit quantitas affirmativa, at versus dextram si q sit negativa. Atque idem de ipsis AG & BR intelligatur, quæ ad contrarias itidem partes duci debent, si modò valores ipsarum r & s prodeant negativi. Denique Centro E & Radio EC = t describatur Circulus CKκc, qui Parabolam in totidem secabit punctis, quot sunt Æquationis datæ Radices reales. Etenim à punctis istis C, K, &c. ducantur CP, Kπ, &c. ipsi ST parallelæ, & ad rectam GR (si opus est productam) terminatæ, eritque harum quævis x, seu Æquationis datæ Radix quæsita; eæ scilicet ad dextram jacentes erunt Radices affirmativæ, quæ verò ad sinistram sunt positæ erunt Radices negativæ. Punctum contactûs, siquod fuerit, hic sumitur pro intersectionis punctis duobus ad invicem vicinissimis.

Inter Æquationes Cubicas & Biquadraticas ita constructas hoc tantùm intercedit discriminis, quòd in prioribus, ob terminum ultimum in præcedente Æquatione deficientem, semper sit $p^2 - q^2 - s^2 + t^2 = 0$, sive $t = \sqrt{s^2 + q^2 - p^2}$. Igitur Centro E & Radio EB $(= \sqrt{BRq + (ERq) STq - VSq})$ descripto Circulo CKκc, Radicum una CP in priori constructione in nihilum abit.

Hæc autem demonstrantur ad modum sequentem. Manentibus jam constructis, & productâ CP, si opus est, donec secat AM in H, erit CH Ordinata Parabolæ ad Diametrum

metrum AH, & proinde $CHq = AL \times AH = AH$, ob $AL = 1$. At $CH = CP + AG$, & $AH = GB + BP$, & propterea $CPq + 2\,AG \times CP + AGq = GB + BP$; sed ob naturam Parabolæ erit $AGq = GB$, unde $CPq + 2\,AG \times CP = BP$. Jam à puncto C ad ipsam BP demittatur normas CD, quæ occurrat etiam ipsi EI, ad BP actæ parallelæ, in puncto I. Propter similia Triangula CDP & TVS, erit $DP = \frac{VS \times CP}{ST}$ & $CD = \frac{VT \times CP}{ST}$, & proinde $CPq + 2AG \times CP = BP = DP + BD = \frac{VS \times CP}{ST} + BR - IE$, sive $CPq + 2AG \times CP - \frac{VS}{ST}CP - BR = -IE$. Ast $IEq = CEq - CIq = CEq - CDq - VTq - 2CD \times VT = CEq - \frac{VTq \times CPq}{STq} - VTq - \frac{2VTq \times CP}{ST} =$ (ob $VTq = STq - SVq$) $CEq - CPq + \frac{SVq}{STq}CPq - STq + SVq - 2ST \times CP + \frac{2SVq}{ST}CP$, quæ igitur æqualis erit Quadrato ex Latere $CPq + 2AG \times CP - \frac{VS}{ST}CP - BR$. Atque hæc Æquatio ad terminos p, q, r, s, t revocata ipsissima fit Æquatio proposita.

Hinc liquet, quòd eadem quævis Æquatio Biquadratica innumeras per Parabolam constructiones sortiri possit, pro indefinito valore quantitatis istius, quam ad arbitrium assumi posse jam diximus. Sed casus est simplicissimus faciendo $VS = p = 0$, & migrat constructio, si rem ipsam spectes, in vulgarem istam, in qua Radicum repræsentatrices rectæ CP, &c. sunt ad Axem perpendiculares. Æquatio autem fit

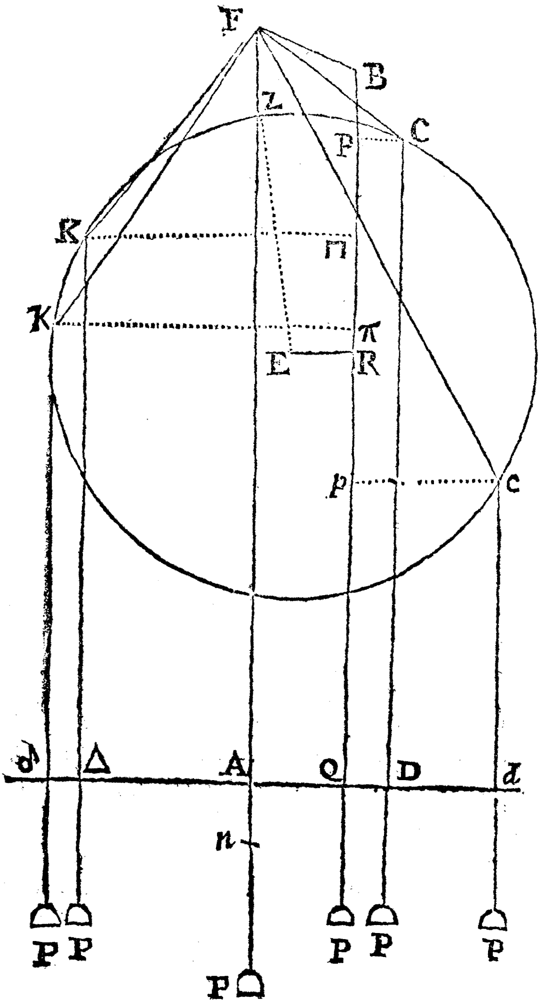
$$x^4 = \begin{matrix} -4rx^3 \\ +2s \\ -1 \end{matrix} \begin{matrix} -4r^2x^2 \\ -2q \\ \end{matrix} \begin{matrix} +4rsx \\ \\ \end{matrix} \begin{matrix} -q^2 \\ -s^2 \\ +t^2 \end{matrix},$$

quæ facile construitur ut supra.

§ 4. Sed ne Parabolæ descriptio Organica difficilis nimium videatur, in promptu est Artificium quoddam Mechanicum, ope Fili penduli pondere instructi peractum, cujus auxilio quam exactissime & facillime Æquatio novissima construi potest, & proinde Æquationum quarumcunque Cubicarum & Biquadraticarum Radices inveniri; idque sine ullo linearum ductu nisi Rectarum & Circuli. Constructio autem, quam appellare libet *Mechanicam*, est ad hunc modum.

Contra Parietem erectum, vel planum aliud quodvis Horizonti perpendiculare, ad punctum aliquod F suspendatur filum tenuissimum & flexile FP; pondere quovis P ad extremitatem P appenso. In hoc filo notetur punctum aliquod N, à puncto suspensionis F satis remotum; vel filo parvulus, si id mavis, innectatur Nodus N. Et sumpta utcunque NO pro Unitate, ad punctum medium A ducatur (in plano prædicto) recta AQ Horizonti parallela, & utrinque quantum satis producta. Hisce generaliter paratis, pro particulari jam applicatione fac AQ = r; ipsis q, r, s, t, ut sæpius inculcatum, vel Arithmeticè vel Geometricè, pro datæ cujusvis Æquationis

ex-



exigentia, in Æquatione novissima priùs determinatis. Tunc Acu vel Stylo tenuissimo, aut etiam cuspide Circini admodum gracili, flectatur filum à loco suo ad punctum quoddam B, ita ut punctum N cadat in novissime invento puncto Q. In BQ ab isto B capiatur BR = s, & in R ad ipsam BR perpendicularis erigatur ER = q. Verùm enimverò istæ AQ, BR, RE ad contrarias partes ab earum initiis cadere debent, si fortè valores ipsarum r, s, q prodeant negativi. Denique in puncto invento E figatur Circini crus unum, &, ad distantiam EZ = t extentum, agatur crus alterum in orbem, secumque circumducat filum FZP. Hac fili circulatione pondus P nunc ascendet nunc descendet motu reciproco, ut & Nodus N nunc supra rectam AQ extabit, nunc verò infra eandem deprimetur. Quoties autem reperietur Nodus ille N in ipsa AQ, puta in punctis D, d, Δ, ♂, abscindet is rectas DQ, dQ, ΔQ, ♂Q.

quæ erunt Æquationis datæ Radices omnes reales; hæ nempe ad dextram erunt Radices affirmativæ, illæ verò ad sinistram Radices negativæ. Demonstratio est manifesta ex præcedentibus, habita tantùm ratione Parabolæ per puncta B, C, c, $\varkappa$, K transeuntis. Nam posito F foco Parabolæ, (cujus distantia à Vertice ast $\frac{1}{4}$ ON,) notum est quod lineæ omnes ut FB + BQ, FC + CD, &c, eandem ubique conficiant summam.

Atque ex principiis hic positis proclive erit Instrumentum haud inconcinnum & quantumvis accuratum fabricari, cujus beneficio hujusmodi Æquationum quarumcunque Radices nullo fere negotio inveniri possint, & præ oculis exhiberi. Hoc autem quilibet, si id Curæ sit, variis modis pro ingenio suo efficere potest, & de his jam satis.

III. *Æquationum quarundam Potestatis tertiæ, quintæ, septimæ, nonæ, & superiorum, ad infinitum usque pergendo, in terminis finitis, ad instar Regularum pro Cubicis quæ vocantur* Cardani, *Resolutio Analytica.*

Per Ab. De Moivre, R. S. S.

Sit n Numerus quicunque, y quantitas incognita, sive Æquationis Radix quæsita, sitque a quantitas quævis omnino cognita, sive ut vocant Homogeneum Comparationis: Atque horum inter se relatio exprimatur per Æquationem

$$ny + \frac{nn-1}{2 \times 3} ny^3 + \frac{nn-1}{2 \times 3} \times \frac{nn-9}{4 \times 5} ny^5 + \frac{nn-1}{2 \times 3} \times \frac{nn-9}{4 \times 5} \times \frac{nn-25}{6 \times 7} ny^7, \text{ \&c.} = a$$

Ex